

# Simultaneous Determination of Thermophysical Properties Using a Thermistor, Part 2: Experiment

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A newly proposed thermistor technique for the simultaneous estimation of thermal conductivity and thermal diffusivity is checked for its accuracy and applicability. Ethanol, toluene, pure water, and mercury, all at 298.15 ± 0.1 K, were selected as target materials. In addition, thermal diffusivity (or thermal capacity) of a powder (bentonite kunigel V1) was also estimated, whereas its thermal conductivity was taken from previously published research. It was found that for both liquids and powder the present method can estimate thermophysical properties with high accuracy. In general, it can determine thermal conductivity and thermal diffusivity within 2 and 6%, respectively. Comparison of thermal properties of liquids measured here with referenced and recently updated data show very good agreement with an accuracy of 2% for thermal conductivity and thermal diffusivity, with the exception of the thermal conductivity of the mercury, found to be accurate to 3.4%.

## Nomenclature

$A$	= dimensionless coefficient in Eq. (12)
$a$	= coefficient in Eq. (13), K
$B$	= dimensionless coefficient in Eq. (12)
$b$	= coefficient in Eq. (13), K
$C_p$	= specific heat of the sample, J/kg · K
$D$	= parameter in Eq. (1), K
$Fo$	= dimensionless time Fourier number
$Gr$	= Grashof number, $[g\beta(T_{gs} - T_{\infty}) \cdot H^3]/\nu_f^2$
$g$	= gravity acceleration, 9.81, m/s <sup>2</sup>
$H$	= characteristic length, $r_c - r_{th}$ , m
$I$	= current supplied to the chip, A
$L_{ch}$	= length of the chip, m
$L_w$	= immersion depth of thermistor, m
$n$	= number of time steps
$n_b$	= porosity of bentonite powder, $1 - \rho_b/\rho_s$
$Pr$	= Prandtl number, $\nu_f/\alpha_f$
$Q$	= heat generated in the chip, W
$q_v$	= heat generation rate per unit time and volume inside the chip, W/m <sup>3</sup>
$R$	= resistance, $\Omega$
$Ra$	= Rayleigh number, $[g\beta(T_{gs} - T_{\infty}) \cdot H^3]/\nu_f\alpha_f$
$R_1, R_2$	= resistances, $\Omega$
$r_c$	= radius of the sample's container, m
$r_{ch}$	= radius of the chip, m
$r_{th}$	= radius of the thermistor, m
$Sr$	= water saturation degree of bentonite powder, $\rho_d \cdot w/\rho_w \cdot n_b$ , %
$T$	= absolute temperature, K

$T_{gs}$	= surface temperature of the glass covering the thermistor chip, K
$T_{\infty}$	= temperature of the quiescent fluid, K
$t$	= time, s
$V$	= voltage supplied to the chip, V
$w$	= water content in bentonite powder, %
$\alpha$	= thermal diffusivity, m <sup>2</sup> /s
$\beta$	= expansion coefficient, $-(1/\rho_f)(\partial\rho_f/\partial T_{\infty})_p$ , K <sup>-1</sup>
$\Delta R$	= resistance absolute error, $\Omega$
$\Delta T$	= absolute temperature error, K; temperature difference, $T_{gs} - T_{\infty}$ , K
$\delta$	= relative error, %
$\theta$	= dimensionless temperature
$\lambda$	= thermal conductivity, W/m · K
$\nu$	= kinematic viscosity, m <sup>2</sup> /s
$\rho$	= density of the sample, kg/m <sup>3</sup>
$\rho_b$	= density of bentonite powder, kg/m <sup>3</sup>
$\rho_s$	= true density of bentonite (no voids), 2700, kg/m <sup>3</sup>
$\rho_w$	= density of water, 1000, kg/m <sup>3</sup>
$\sigma$	= standard deviation, %

## Subscripts

ch	= chip
f	= fluid
meas	= measured
exp	= experimental
v	= volume averaged

## Introduction

THE estimation of thermophysical properties from measured temperature data is known as a parameter estimation method. Different sensors can be used to measure temperature, for instance, thermocouples, thermoresistance, and thermistors. In the conventional thermistor technique, the temperature rise within the thermistor is kept constant.<sup>1–4</sup> This requires a variation of the bead power dissipation during the measurement, based on an instantaneous step change.<sup>5</sup> The estimation of thermal conductivity is extracted from steady state, which requires collection of the data for at least 5 s to enable extrapolation to infinity to get the steady state. This means that these data are affected by natural convection effects in the case of liquids, conduction along the lead wire, and the return of a thermal wave after it reaches the boundary of the sample's container.

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To avoid these problems, an improved version of the conventional thermistor technique is proposed by the authors.<sup>6</sup> Using this technique, the authors have succeeded in measuring thermal conductivity of liquid and powder bentonite kunigel V1 and the mixture bentonite-silica sand.<sup>7</sup> The accuracy in both measurements was found to be good, and, for the bentonite, the data correlate very well with all of the best available correlations related to porous materials and mixtures.

In these two studies, thermal conductivities were estimated under the assumption that thermal capacity is known parameter, which makes the estimation procedure much easier than that for the simultaneous estimation of  $\lambda$  and  $\alpha$  (or  $\rho C_p$ ).

To further improve this technique, the authors have made some modifications related to the method of estimation, so that it can be used for the simultaneous estimations of thermal conductivity and thermal diffusivity. The details of the numerical method, as well as the procedure of estimation of  $\lambda$  and  $\alpha$ , are given in a separate paper.<sup>8</sup>

To show the accuracy of this new thermistor technique, simultaneous estimation of thermal conductivity and thermal diffusivity of ethanol, toluene, pure water, and mercury are presented. For the powder bentonite, and for selected samples with porosity 0.3 and different water saturation degrees, specific heats are estimated and compared to those predicted by the use of a relation derived by Japan Nuclear Cycle Research Institute (JNC)<sup>9</sup> for bentonite.

Because it is a newly proposed method, and for enhanced clarity and understanding, mercury is selected as a target material with which to show all of the steps to follow until thermal conductivity and thermal diffusivity are estimated. For the same purpose, the experimental procedure is given in detail, along with the different sources of errors in it.

## Experiment

The accurate measurement of thermal conductivity and thermal diffusivity is a difficult task because, being transport properties, the thermal conductivity and thermal diffusivity are more difficult to measure than thermodynamic properties such as density and specific heat. Thermophysical properties are not measured directly, but are instead extracted from the temperature–time history of the material. Because the temperature itself is also measured indirectly through information given by the variation of physical properties, such as the resistance of the thermistor, it is expected that the main error in the measurement of thermophysical properties, especially thermal diffusivity, would come from the error in the measurement of temperature.

The thermistor used in the experiments is a negative temperature coefficient (NTC) type thermistor, specifically a PSB-S7 43 (Shibaura Electric Company, Inc.) and is manufactured mainly from an Ni, Mn, and Co oxide system.<sup>10</sup> Its electrical resistance falls sharply with rises in temperature. The relation between the resistance and the temperature is expressed as

$$R_1 = R_2 \exp\{D[1/T_1 - 1/T_2]\} \quad (1)$$

where  $R_1$  and  $R_2$  are the resistances at absolute temperatures  $T_1$  and  $T_2$ , respectively, and  $D$  (Kelvin), is a constant with values tabulated for each type of NTC thermistor depending on the range of temperatures being measured. As an example, the constant  $D$  takes the values of 3384.27 K in the temperature range from 293.15 to 298.15 K. The resistance of the thermistor at 293.15 K is 13.033 k $\Omega$ , and the temperature in the range 293.15–298.15 K is then calculated by the use of Eq. (1) as

$$T = 1/[(1/293.15) + (1/3384.27) \ln(R/13.033)] \quad (2)$$

where  $R$  (kilo-ohms) is the measured resistance of the thermistor at  $T$  (Kelvin). Equation (2) shows that a change in temperature of 0.1 K leads to a change in resistance of about 50  $\Omega$ . The thermistor used in the present study maintains its high sensitivity to temperature from

223.15 to 523.15 K, and so the thermal conductivity over a wide range of temperatures can be estimated by the present method.

## Method of Measurement

The information needed from the experiments are the temperature–time history, the profile of heat generated in the thermistor, and the temperature of sample before the start of heating.

Figure 1 shows the measurement circuit where the components are arranged in series. The voltage supplied with a dc power generation was selected depending on the nature of the target material. A digital multimeter 7555 Yokogawa with a resolution of 1 mV measures the voltage drop of the thermistor and the variable resistance. The type of the A/D converter is a Canopus ADME-1201 12 bit. The data logger acquisition receives the data and sends them to the personal computer for treatment, where they are managed with Microsoft Excel.

The heat generation rates can be set arbitrarily in the present method, but a constant heat generation was selected because it is suitable for accurate measurement of thermophysical properties. This can be attained by adjustment of the variable resistance before the start of heating to be equal to that of the thermistor.

The experiment is repeated five times, resulting in five sets of data. The fluctuations in voltage and current output of the power supply from run to run were found to be very small, almost equal to 0.1%. For liquids, the thermistor was inserted into glass bottles of 1-cm diam and 2.5-cm height, containing liquids (pure water, ethanol, toluene, glycerol, and mercury). The bottle is covered by polyurethane and positioned in a constant temperature box. A cooling time of 5 min was found sufficient to eliminate heating interference from one experiment to the next. The repeatability was found to be very good. The final experimental data produced were the averaged value of the five measurements. For mercury, a power of about 8.2 mW was supplied, and the resulting temperature rise of thermistor chip was about 1.7 K. Using approximately the same heat generation, we obtained a temperature rise of about 4.2 K for glycerol whereas for pure water, it was around 3.5 K. Toluene and ethanol experienced heat generations of approximately 4.3 and 5.9 mW, respectively. The temperature rise was equal to 3.2 and 2.8 K for toluene and ethanol, respectively. The specimen powder (bentonite kunigel V1), was also measured. This material consists of granular Na-bentonite, which contains about 50% montmorillonite, beside the minerals quartz, feldspar, calcite, pyrite, etc. The mineral composition can be found in the study of Fujita et al.<sup>11</sup> The details of the experiments are given in a recently published paper.<sup>7</sup> The shape and the volume of the samples are cylindrical and equal to  $\pi$  cm<sup>3</sup>. Samples with different porosities and water contents were tested; however, in the present study, only the results for the samples with a porosity equal 0.3 are presented. The water saturation degrees are  $S_r = 0, 13.5, 27, 40$ , and 94%, respectively. All of the measurements were conducted at room temperature,  $298.15 \pm 0.1$  K. The data were measured and recorded with a frequency of data sampling equal to 5 kHz. The thermal conductivity and thermal diffusivity of the liquids and the bentonite powders are evaluated at the arithmetic mean temperature between the glass surface temperature and the temperature of the sample before heating.

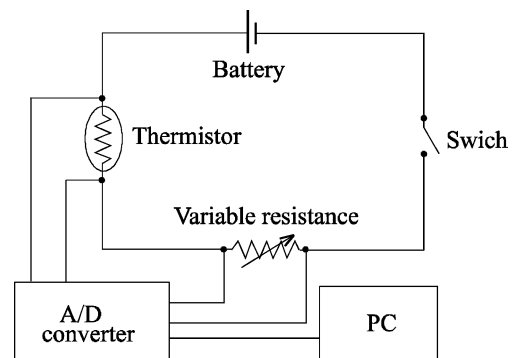


Fig. 1 Measurement circuit.

## Measurement Errors

In every measurement, errors exist whether they are due to the technique of measurement, human error, or the instrumentation used. Because the estimation of thermophysical properties uses temperature measurements, the present method can be applied only if these measurements are assumed to be free from bias errors. Without bias errors, we still have random errors in the temperature measurement. This assumes that the error is due to the drift of the thermistor from its original characteristics, the noise in the 12-bit A/D converter, the errors in the voltmeter, and the frequency of data sampling.

### Thermistor Drift

To evaluate the stability of the NTC thermistor, examination of the resistance change at a given temperature with time or after cycling is required. The aging characteristics in terms of percent resistance changes with time in air at 373.15~423.15 K for the NTC thermistors, as observed by the manufacturer. However, the observation is not a general rule because the drift resistance depends on the chemical composition, the crystal structure, and the thermal history of the ceramics.<sup>12–16</sup> The present thermistor is composed of the oxides of (Ni, Mn, and Co) and the stability of such a device was found to depend not only on the grain size and microstructure, but also on the content of nickel.<sup>17,18</sup>

The drift of the resistance from its nominal value will lead to a temperature measurement error. The absolute temperature error at 298.15 K is estimated to be

$$\Delta T = (\Delta R/R)[(298.15)^2/D_{298.15}] \approx 0.05 \text{ K} \quad (3)$$

Because the coefficients  $D$  are tabulated<sup>10</sup> every 5 K, the error has a bell shape, and the maximum occurs at the midinterval. As an example, the maximum error in temperature measurement by the use of the  $D$  coefficient at 298.15 K will occur at 295.65 K, for which no value of  $D$  is available. This value is actually the maximum error for any range of temperature measurement due to the calibration or drift of the thermistor resistance. It is given by Eq. (4) as equal to

$$\Delta T_{\max} = [1/(D_{298.15}/298.15^2)][(\Delta R/R) + (D_{298.15} - D_{295.65})(1/295.65 - 1/298.15)] \approx 0.052 \text{ K} \quad (4)$$

The maximum temperature error that can be done due to the drift of the resistance of the thermistor is equal to 0.052 K.

### 12-Bit A/D Converter Error

A/D converter errors may affect the uncertainty in set-point calculations. The following A/D converter errors are described: the least-significant-bit (LSB) error, the linearity error, the gain error, the offset error, the nonmonotonicity, and the missing code. The LSB error is present in all A/D converters. The maximum LSB error is defined as one-half of the LSB. The 12-bit A/D Converter has a 0.5 part in  $2^{12}$ , or 0.0122% of full-scale input voltage as its maximum LSB error. The other errors are given also in LSB. The total error is equal to 9 LSB, or 0.2196% of full-scale voltage. The total percent full-scale voltage error multiplied by the input voltage range provides the maximum voltage error ( $5 \text{ V} \times 0.002196 = 11 \text{ mV}$ ).

### Digital Voltmeter

As mentioned before, the voltmeter used in the experimentations is a digital multimeter 7555 Yokogawa with a resolution of 1 mV. The maximum error that can occur is then 1 mV.

### Frequency of Data Sampling

When an A/D converter samples the analog signal at fixed time intervals, each sample is represented by a binary number. Between sampling intervals, the value of the analog signal is unknown to the system. If the analog signal changes significantly between samples, the digital signal will not adequately represent it. In this case, high-frequency components of the analog signal are falsely represented as lower-frequency components in the digital signal. This false representation of frequencies is called aliasing, and it has the potential

to degrade input signals and to lead to false estimation of thermophysical properties. One way to address aliasing is to sample the analog signal extremely quickly, in which case aliasing still occurs, but the power of aliased frequencies is extremely low.

To determine both thermal conductivity and thermal diffusivity by the use of an extremely fast sampling frequency, a selection of frequency is based on the intrinsic time constant of the chip, which is equal to 63.2% of the typical temperature rise within the chip.

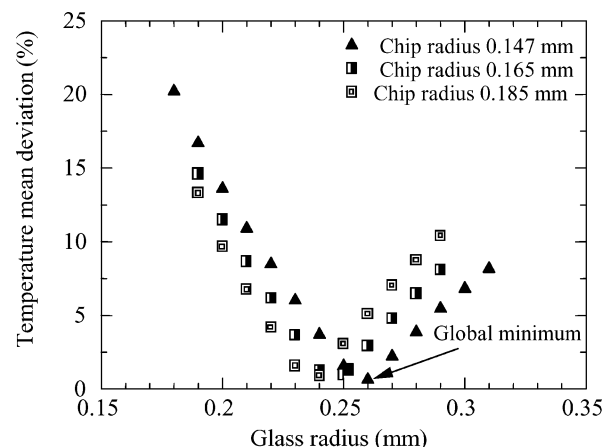
The intrinsic time constant of the chip is almost equal to 0.2 ms. This supposes that the frequency of the sampling has to be at least equal to 5 kHz to record correctly the variation of temperature within the chip. For high thermal conductivity/diffusivity materials, this frequency needs to be increased further. It was selected as equal 10 kHz for the measurement of mercury.

## Calibration Method Using Glycerol as Standard Material

For the calibration of the thermistor probe, glycerol was used as standard material because of its high viscosity, which eliminates natural convection effects. The thermal conductivity of glycerol at  $298.15 \pm 0.1 \text{ K}$  is given as  $0.297 \text{ W/m} \cdot \text{K}$  by Lide,<sup>19</sup> whereas it is tabulated in the *JSME Data Book*<sup>20</sup> as equal to  $0.288 \text{ W/m} \cdot \text{K}$ . The  $0.297 \text{ W/m} \cdot \text{K}$  value was used here. The thermophysical properties of the thermistor used in the measurements as given by the manufacturer (Shibaura Electric Company, Inc.) are presented in Table 1. The thermistor is rectangular parallelepiped and its estimated volume is about  $0.0135 \times 10^{-9} \text{ m}^3$ . In the simulated model,<sup>16</sup> the chip is cylindrical; therefore, the radius or the length must be estimated. From microscope observations, the length of the chip was found to be approximately equal to 0.2 mm. The length of the glass protection was 1.0 mm, and this length was used as the length of the simulated cylindrical glass protection. The remaining parameters to be estimated are the radii of the chip and the glass protection. For a set value of chip radius, the glass protection radius varies, and a set of curves was obtained as shown in Fig. 2. Each curve represents the variation of the standard deviation  $\sigma$  (%) of the temperature vs the radius of the glass protection for a specified value of thermistor chip radius. All of the curves pass through a minimum, and the lowest value of the mean deviation among all of the curves was chosen as the best

**Table 1** Dimensions and estimated thermophysical properties of the thermistor probe

Material	Radius, mm	Length, mm	$\lambda$ , W/m · K	$\alpha \times 10^6$ , m <sup>2</sup> /s
Dumet wire	0.05	26	172	42.219
Thermistor chip	0.14	0.2	20	10.256
Insulating material	Thickness	0.02	0.1	0.113
Glass protection	0.26	1.0	0.85	0.554



**Fig. 2** Calibration of the thermistor probe.

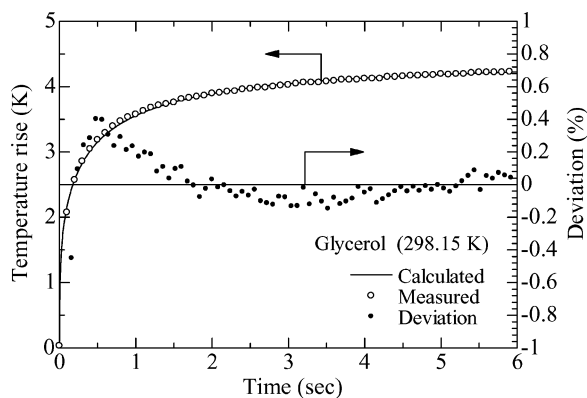


Fig. 3 Measured and calculated temperature histories of glycerol used as standard material.

calibration of the thermistor. The dimensions for each component of the simulated thermistor are listed in Table 1. The lead wires were covered with paint to isolate them from leaking electricity when mercury was measured.

Figure 3 shows the measured and calculated temperature histories<sup>8</sup> for glycerol at  $298.15 \pm 0.1$  K. The deviation between the two curves is very small, showing the high accuracy of the numerical model in the prediction of the temperature history of the thermistor probe.

### Results and Discussion

From Fig. 2, the radius of the chip is estimated as equal to 0.14 mm. By the use of this value, it can be shown from the numerical results<sup>8</sup> that for mercury within time of measurement of thermophysical properties (ending at 0.1 s), a sample with a distance from the thermistor probe to the boundary in the axial direction equal to 6.45 mm is enough to ensure a measurement safe from the boundary errors. However, for glycerol, a much bigger sample is needed because the measurements have to be done with enough recorded data (which were situated between 0.2 and 1.8 s) to ensure an accurate calibration. The axial distance should be at least 10.5 mm. In the present experiment, the length of the glass bottle is 2.5 cm and its radius 0.5 cm, which implies that the requirements are fulfilled.

Thermal conductivity of bentonite is higher than that of glycerol and lower than mercury. This means that, for bentonite kunigel-V1, the measurements were also safe from the boundary errors with 2-cm radius and 1-cm height.<sup>7</sup> The dimensions of the sample selected are, therefore, suitable for accurate measurement and quick spreading of moisture in the sample.

The accuracy of the present method was checked with materials with known thermophysical properties. Ethanol (99.5% purity), pure water, toluene (99.5% purity), and mercury, all at  $298.15 \pm 0.1$  K, were selected for this purpose. During experiments, the heat generated in the thermistor was selected to provide very good reproducibility, which lead to the smallest 95% confidence interval possible.

In the conventional thermistor techniques, estimation of thermal conductivity is taken from data extrapolated to infinity.<sup>2–4</sup> The time window for these data is situated between 5 and 8 s, depending on the nature of the target material. For liquids, the onset of natural convection effects is, therefore, present in the collected data. To reduce the Rayleigh number  $Ra$ , defined as  $Ra = Gr \cdot Pr$ , the temperature rise  $\Delta T = T_{gs} - T_{\infty}$  can be reduced. However, this leads to a decrease in the signal to noise ratio. As a consequence, the 95% confidence interval increases significantly and affects the accuracy of the measured thermal conductivity and thermal diffusivity. In the present method, transient-state data are used, which avoids completely the onset of natural convection effects, even for liquids with high Prandtl numbers.

In Figs. 4–7 we plot the calculated and measured temperature histories for pure water, ethanol, toluene, and mercury, respectively.

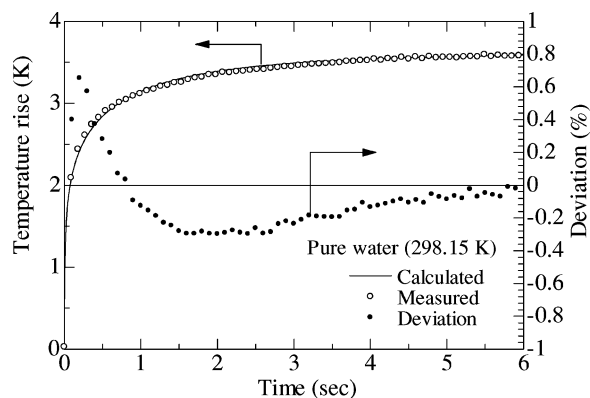


Fig. 4 Calculated and measured temperature histories for pure water.

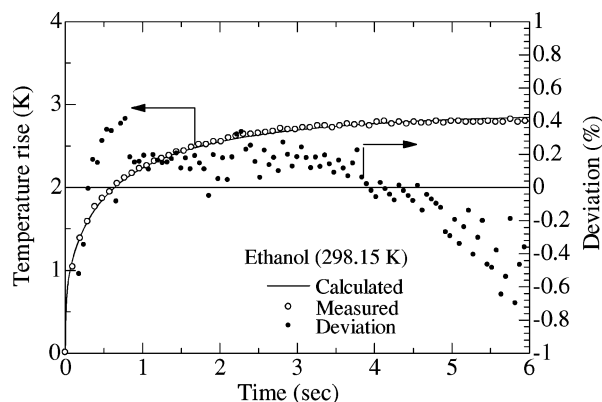


Fig. 5 Calculated and measured temperature histories for ethanol.

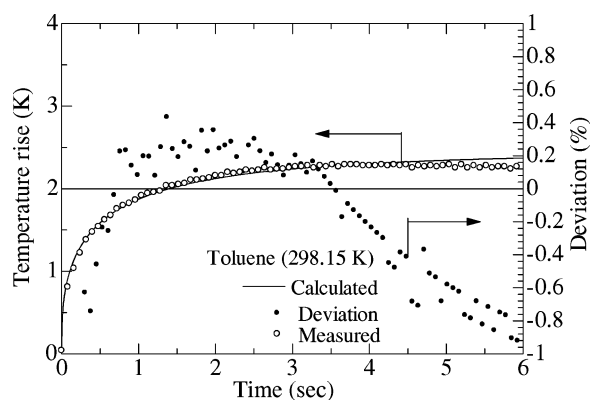


Fig. 6 Calculated and measured temperature histories for toluene.

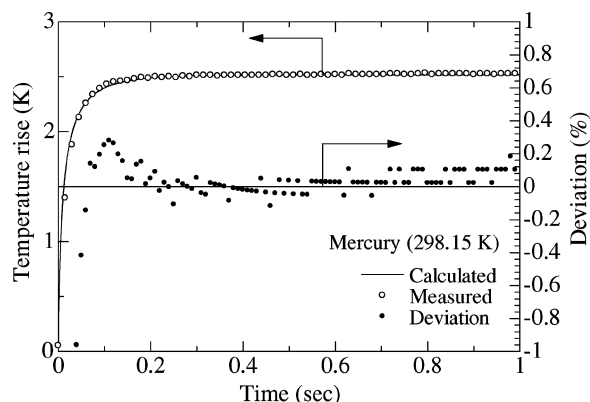


Fig. 7 Calculated and measured temperature histories for mercury.

The onset of natural convection can be observed when the deviation between the measured and calculated temperature histories is greater than  $-0.5\%$ , which means that the measured temperature starts to decrease with time. The onset of natural convection starts for toluene before ethanol, as shown in Figs. 5 and 6. This is predicted from the respective values of Rayleigh number:  $Ra = 23,270$  for toluene ( $\Delta T \approx 1$  K), and  $Ra = 12,953$  for ethanol ( $\Delta T \approx 1.3$  K). However, for pure water, and even with a higher heat generation, the onset of natural convection cannot be observed within the first 5 s with a Rayleigh number  $Ra = 5030$  ( $\Delta T \approx 2.3$  K). In general, for a time window selected between 0.5 and 2 s, no natural convection effects are observed. This shows that our present estimation method is free from this systematic error, which is not the case for the conventional thermistor method.

For pure water, thermal conductivity and thermal diffusivity based on five experiments were found equal to  $0.607 \text{ W/m} \cdot \text{K}$  and  $0.147 \times 10^{-6} \text{ m}^2/\text{s}$ , respectively. The 95% confidence intervals were very small, about 0.16, and 2.7%, respectively. This is due to the high signal to noise ratio obtained by the increase in temperature step. For toluene, thermal conductivity and thermal diffusivity were estimated to equal  $0.1308 \text{ W/m} \cdot \text{K}$  and  $0.0882 \times 10^{-6} \text{ m}^2/\text{s}$ , respectively. The 95% confidence interval was higher than for pure water equal to 1.8% for thermal conductivity and 4.2% for thermal diffusivity. For ethanol, the confidence interval was equal to 0.9 and 3.9% for thermal conductivity and thermal diffusivity, respectively. Thermal conductivity was found to equal  $0.167 \text{ W/m} \cdot \text{K}$ , and thermal diffusivity  $0.0869 \times 10^{-6} \text{ m}^2/\text{s}$ . The low signal-to-noise ratio in the measurements of ethanol and toluene can be noticed in Figs. 5 and 6, in which the scattering of the deviations are much more significant than in pure water. However, in the selected time range, the deviations are, in general, less than  $\pm 0.5\%$ . The thermal conductivity of mercury is estimated as  $8.12 \text{ W/m} \cdot \text{K}$  at  $298.15 \pm 0.1 \text{ K}$ , whereas the thermal diffusivity is  $3.4517 \times 10^{-6} \text{ m}^2/\text{s}$ . From Fig. 7, also note that the present method succeeded in predicting the temperature history of the mercury and that the deviation was lower than 0.4% almost everywhere. There are few measurements of this property in the literature. In a recently published paper, Nagai et al.<sup>21</sup> measured the thermal conductivity of mercury under microgravity conditions using the hot-disk method. Their estimated value was equal to  $7.85 \text{ W/m} \cdot \text{K}$  at atmospheric pressure and for a temperature of  $295.15 \text{ K}$ . In Ref. 22, thermal conductivity of mercury at  $298.15 \text{ K}$  is quoted as equal  $8.24 \text{ W/m} \cdot \text{K}$ , whereas it is equal to  $8.44 \text{ W/m} \cdot \text{K}$  at  $293.15 \text{ K}$  in Ref. 23. The Rayleigh number based on a temperature rise  $\Delta T \approx 1 \text{ K}$  is found to be  $Ra = 370$ .

As examples of estimations of  $\lambda$  and  $\alpha$ , results for ethanol, toluene, pure water, and mercury are presented in Table 2. These results correspond to the measurements shown in Figs. 4–7. Observation of experimental temperature history for pure water shows that the time range used to extract the information where a linear equation can be approximated is situated between 0.2 and 2 s. However, for toluene and ethanol, this time is situated between 0.3 and 1.5 s, and, for mercury, the time range is situated between 20 and 100 ms.

Following the method of estimation given in Part 1 of this paper,<sup>8</sup> after the convergence is reached, coefficients  $A$  and  $B$  are obtained from the fitting equation of the calculated volume-averaged temperature rise of the thermistor, which takes the form  $\theta_v = A + B \ln Fo$ . The corresponding coefficients  $a$  and  $b$  for the best fit of the experimental data by the use of the equation  $T_v = a + b \ln t$  are also extracted. Therefore, the estimated values for  $\lambda$  and  $\alpha$  are given by Eqs. (5) and (6), respectively, as

$$\lambda = q_v r_{ch}^2 (B/b) = VI/\pi L_{ch} \cdot B/b \quad (5)$$

$$\alpha = r_{ch}^2 \exp(a/b - A/B) \quad (6)$$

Note that these coefficients correspond to a timescale in milliseconds; therefore, the value obtained for thermal diffusivity must be multiplied by 1000 to obtain a value in square millimeters per second. Figures 8 and 9 show the calculated and measured temperatures and their corresponding best fits for the case of mercury.

To the best of our knowledge, few researchers used thermistor technique to estimate thermal properties of porous materials. Woodbury and Thomas<sup>24</sup> were among them. Their study was concerned primarily with moisture laden glass fiber insulating materials with a range of thermal conductivity situated between 0.03 and  $0.12 \text{ W/m} \cdot \text{K}$  for completely dried and saturated samples, respectively.

Thermal conductivities of pure bentonite kunigel-V1, and the mixtures bentonite–silica sand have been measured by the authors<sup>7</sup> using the present method. This material, produced in Japan, was measured by very few researchers.<sup>7,9,11,25–27</sup> Data for samples with different porosity, water contents, and sand volumes were found to correlate well with all known available correlations of porous materials, in particular with the correlation of Sakashita and Kumada,<sup>28</sup> which was derived specifically for bentonite. In addition, the scattering of the previously reported data<sup>9,11,29</sup> is larger than that of the data measured by the authors.

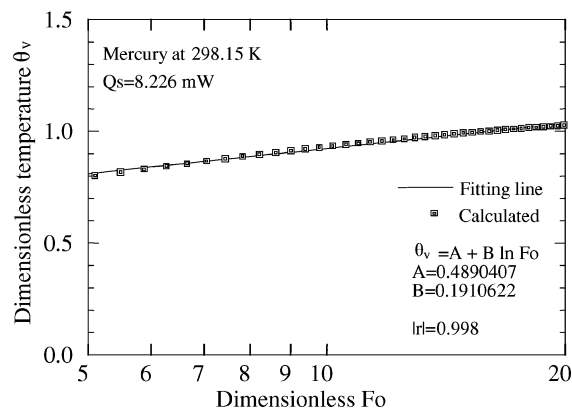


Fig. 8 Calculated dimensionless temperature history and its best fit for mercury.

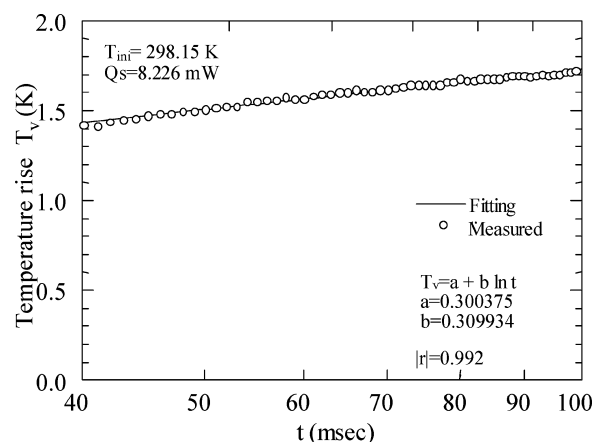


Fig. 9 Measured temperature history and its best fit for mercury.

Table 2 Coefficients  $A$ ,  $B$ ,  $a$ ,  $b$ , and estimation of  $\lambda$  and  $\alpha$  for ethanol, toluene, pure water, and mercury

Material	$A$	$B$	$a$	$b$	$Q$ , mW	$\lambda$ , W/m · K	$\alpha \times 10^6$ , m <sup>2</sup> /sec
Ethanol	0.08187791	0.02848025	-1.863685	0.7327843	5.919	0.167	0.0869
Toluene	0.07274345	0.02767955	-1.845278	0.6648734	4.328	0.131	0.0882
Pure water	0.1087041	0.0239827	-0.167404	0.4702469	8.226	0.607	0.1470
Mercury	0.4890407	0.1910622	0.300375	0.309934	8.226	8.117	4.3810

In the present study, samples with porosity  $n_b = 0.3$  and different saturation degrees  $Sr$  are selected to check the accuracy of the method in predicting thermal diffusivity, whereas thermal conductivity is taken from the previous study.<sup>7</sup> For the thermal diffusivity of bentonite, very few data are available for comparison; however, specific heat can be used for comparison through the relation given by the Power and Nuclear Cycle Research Institute (PNC),<sup>9</sup> which gives the specific heat of bentonite kunigel-V1 for any water content as

$$C_p = \frac{4.19 \cdot w + 42.6}{100 + w} \quad (7)$$

where  $w$  is the water content given in percent as the ratio of the mass of moisture to the mass of dry bentonite.

In Table 3, a comparison between the present estimations of specific heat and the predictions that use the PNC relation is presented. The density of the sample can easily be measured. Thermal diffusivity and thermal conductivity are known from the simulations. Specific heat can then be calculated with the relation  $C_p = \lambda_{\text{meas}} / (\rho_{\text{meas}} \cdot \alpha_{\text{meas}})$  for each sample.

The results show that the relative error is situated between  $-14\%$  and  $+7.8\%$ . This can be taken as a good estimation.

**Table 3** Comparison of the present data for specific heat and PNC<sup>9</sup> data

$Sr, \%$	PNC <sup>9</sup>		Present estimation	Relative error, %
	$C_p, \text{kJ/kg} \cdot \text{K}$	$C_p, \text{kJ/kg} \cdot \text{K}$		
0	0.426	0.438		+3.0
13.5	0.724	0.781		+7.8
27.4	0.956	1.004		+5.0
40	1.501	1.584		+4.9
94.7	2.256	2.225		-1.4

## Uncertainty Analysis

From Eqs. (5) and (6), the relative errors for  $\lambda$  and  $\alpha$  can be estimated as

$$\frac{\delta \lambda}{\lambda} = \left[ \left( \frac{\delta V}{V} \right)^2 + \left( \frac{\delta I}{I} \right)^2 + \left( \frac{\delta L_{\text{ch}}}{L_{\text{ch}}} \right)^2 + \left( \frac{\delta B}{B} \right)^2 + \left( \frac{\delta b}{b} \right)^2 \right]^{\frac{1}{2}} \quad (8)$$

$$\frac{\delta \alpha}{\alpha} = \left[ \left( \frac{2\delta r_{\text{ch}}}{r_{\text{ch}}} \right)^2 + \left\{ \delta \left( \frac{a}{b} \right) \right\}^2 + \left\{ \delta \left( \frac{A}{B} \right) \right\}^2 \right]^{\frac{1}{2}} \quad (9)$$

The factors in Eqs. (8) and (9) are estimated as follows: Error in the slope of temperature against  $\ln t$  includes the electrical noise and the timing of the voltage measurements. The values of  $\delta b/b$  and  $\delta(a/b)$  are accurate to 1 and 5%, respectively. From numerical solutions,  $\delta B/B$  and  $\delta(A/B)$  are found to equal 0.4 and 0.5%, respectively. These values include the uncertainties of the thermophysical properties and the dimensions of the materials comprising the thermistor. The voltage and the current supplied to the chip are measured by the use of digital multimeters. Here,  $\delta V/V$  and  $\delta I/I$  are less than 0.1%. The volume of the chip was measured by the manufacturer; however, the length or the radius of the thermistor chip are estimated by calibration. Both  $\delta L_{\text{ch}}/L_{\text{ch}}$  and  $\delta r_{\text{ch}}/r_{\text{ch}}$  are found to be accurate to 1.4% by variation of the value of  $r_{\text{ch}}$  and a check of the standard deviation. However, these parameters depend on the accuracy of the volume of thermistor chip given by the manufacturer. For this reason, it is difficult to determine accurately the actual values of  $\delta L_{\text{ch}}/L_{\text{ch}}$  and  $\delta r_{\text{ch}}/r_{\text{ch}}$ . The uncertainty in the present work can be estimated by a comparison between the measured and the referenced values of thermal conductivity and thermal diffusivity. If 1.4% is taken as the actual value of  $\delta L_{\text{ch}}/L_{\text{ch}}$  and  $\delta r_{\text{ch}}/r_{\text{ch}}$ , the total errors in the current method are 2 and 6% for thermal conductivity and thermal diffusivity, respectively.

Table 4<sup>30–32</sup> shows our estimations of thermal conductivity and thermal diffusivity and the most commonly accepted data quoted in

**Table 4** Example of estimation of  $\lambda$  and  $\alpha$  for mercury, pure water, toluene, and ethanol

Thermal conductivity, W/m · K				Thermal diffusivity × 10 <sup>6</sup> , m <sup>2</sup> /sec			
Reference value	Measured value at 298.15±0.1 K	Relative error of λ, %	95% Confidence interval for λ, %	Reference value	Measured value at 298.15±0.1 K	Relative error of α, %	95% Confidence interval for α, %
Mercury							
7.850 <sup>21</sup> (295.15 K)	8.117	+3.4	0.4	—	4.3810	—	3.2
8.247 <sup>22</sup> (298.15 K)	—	−1.6	—	4.3763 <sup>22</sup> (298.15 K)	—	+0.1	—
8.447 <sup>23</sup> (293.15 K)	—	−3.9	—	4.4670 <sup>23</sup> (293.15 K)	—	−1.9	—
Pure water							
0.607 <sup>19</sup> (298.15 K)	0.607	0.0	0.16	—	0.1470	—	2.7
0.608 <sup>30</sup> (300 K)	—	−0.2	—	—	—	—	—
0.610 <sup>20</sup> (300 K)	—	−0.5	—	0.1466 <sup>20</sup> (300 K)	—	+0.3	—
0.607 <sup>31</sup> (298.15 K)	—	0.0	—	—	—	—	—
Toluene							
—	0.131	—	1.8	—	0.0882	—	4.2
0.130 <sup>20</sup> (300 K)	—	−0.8	—	0.0866 <sup>20</sup> (300 K)	—	−1.9	—
0.131 <sup>32</sup> (298.15 K)	—	0.0	—	—	—	—	—
Ethanol							
0.169 <sup>19</sup> (298.15)	0.167	−1.2	0.9	0.0880 <sup>19</sup> (298.15 K)	0.0869	−1.3	3.9
0.166 <sup>30</sup> (300 K)	—	+0.6	—	—	—	—	—
0.166 <sup>20</sup> (300 K)	—	+0.6	—	0.0864 <sup>20</sup> (300 K)	—	+0.6	—

literature. Observations of those data indicate that our estimations for both thermal conductivity and diffusivity are very good and are situated within the predictions of uncertainty. The thermal conductivities of the three ordinary liquids were found to be within  $\pm 1.2\%$  of the references, whereas for mercury, the difference was  $3.4\%$ . The larger difference for mercury is because its thermal conductivity is almost 20 times higher than thermal conductivity of glycerol, which was used to calibrate the thermistor probe.

## Conclusions

Some objectives were assigned before the present research was begun, among them, we can count proposing a method for simultaneous measurements of thermal conductivity and thermal diffusivity. This method can be used for both liquids and powders. It should use small sample sizes. It needs to be robust because it will be used for highly compacted powders. Finally, the measurements need to be finished over a short period of time. The present proposed thermistor technique has successfully fulfilled all of these requirements, as the paper has shown.

More specifically, the improvements can be summarized by the following points:

1) The present proposed thermistor method succeeded to measure with high accuracy thermal conductivity of liquids in a wide range from 0.1 to almost  $9 \text{ W/m} \cdot \text{K}$ , which covers the range of most liquids and powders. Both measured thermal conductivities and thermal diffusivities of the liquids were found to be in very good agreement with the most accepted reference data.

2) Thermal conductivity of bentonite powder was also measured by the authors, and good agreement was found. This means that the current method can measure both liquids and powders with high accuracy.

3) Thermal diffusivities were also measured, and comparisons to the available reference values show good agreement.

4) The thermophysical properties are determined by a small temperature rise in a small sample. In addition, the temperature-time history is selected in the transient-state part; this makes it possible to suppress the natural convection effects for liquids and the redistribution of moisture for powders that include moisture.

In summary, this proposed method is the most accurate and simple to implement technique that has ever been proposed as a point heat source method. It yields thermal conductivity and thermal diffusivity for standard liquids within  $2\%$  of accuracy.

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